# Self-Organization Induced Scale-Free Networks

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What is the underlying mechanism leading to power-law degree distributions of many natural and artificial networks is still at issue. We consider that scale-free networks emerges from self-organizing process, and such a evolving model is introduced in this letter. At each time step, a new node is added to the network and connect to some existing nodes randomly, instead of "preferential attachment" introduced by Barabási and Albert, and then the new node will connect with its neighbors' neighbors at a fixed probability, which is natural to collaboration networks and social networks of acquaintance or other relations between individuals. The simulation results show that those networks generated from our model are scale-free networks with satisfactorily large clustering coefficient.

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The last few years have witnessed a tremendous activity devoted to the characterization and understanding of complex networks [1, 2, 3], which arise in a vast number of natural and artificial systems, such as Internet [4, 5, 6], the World-Wide Web [7, 8], social networks [9, 10, 11, 12], airports network [13, 14], food webs [15, 16], biological interacting networks [17, 18, 19] and so forth. Particularly, much attention has been dedicated recently to the study of scale-free networks, namely, networks that display a power-law degree distributions, $P(k) \propto k^{-\gamma}$, where $k$ is the connectivity (degree) [20]. And various proposals for dynamical evolution of scale-free networks has been introduced. Roughly, these models can be classified into two main scenarios [21]. One is under the mechanism "preferential attachment", which means new vertices are preferentially attached to existing vertices with large number of neighbors [22, 23, 24, 25], and a related scenario is found in the protein duplication model [26]. Another is a balance between a modelled tendency to form hubs against an entropy pressure towards a random networks [27, 28, 29].

However, what is the underlying mechanism leading to power-law degree distributions is still at issue. Differing from the two class of mechanisms mentioned above, we consider that self-organization maybe the fundamental mechanism which leads to power-law distribution of degree, and a model including such a mechanism is introduced in this letter. We think that the node newly added to the network will connect to some existing nodes randomly, not preferentially. For instance, considering the scientist collaboration networks, most of the times, a scholar comes into a new field not because he correlated

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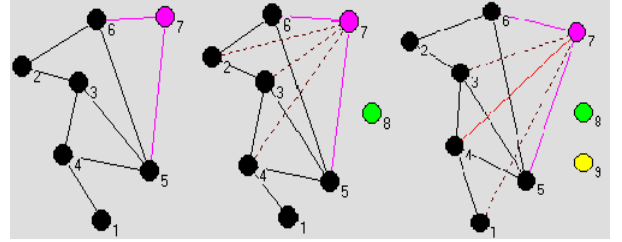†Electronic address: zhutou@ustc.edu

FIG. 1: A schematic representation of the evolving rules for the case $m = 2, G = 2$. At each time step, a new node is added to the network. For each new node, it will be active along $G + 1$ steps. In the first time step(i.e. the step when it was added to the network), it will randomly choose $m$ nodes to be its neighbors. Then, in the following $G$ steps, with probability $p$, this node will connect to the neighbors of the nodes which connected to it at the last step. To make it easier to understand, we draw the figure 1 as a sketch map for the case $m = 2, G = 2$. Assume node 7 connected to two nodes 5 and 6(left), then it will link to the neighbors of 5 and 6, i.e. 2, 3, 4, with probability $p$. Obviously, if some node is common neighbor of 5 and 6, the probability is $2p$. At the same step a new node 8 is added to the network by linking to $m=2$ nodes randomly (omitted in Fig.1) (mid). Suppose node 7 connected to node 4 at last step, then it links to neighbors of 4, i.e. 3, 1, with the same probability $p$ (right). The operation for each node lasts for $G$ steps similarly.

with some famous people in this field , but he read some experts' papers by chance and then connected with them or some of his partners were in this field already. Thus, a relatively new node having linked to some other nodes in the network will connect with its neighbors' neighbors, whose evidences can be found in collaboration and friendship networks. Therefore, we propose an evolving model following the rules:
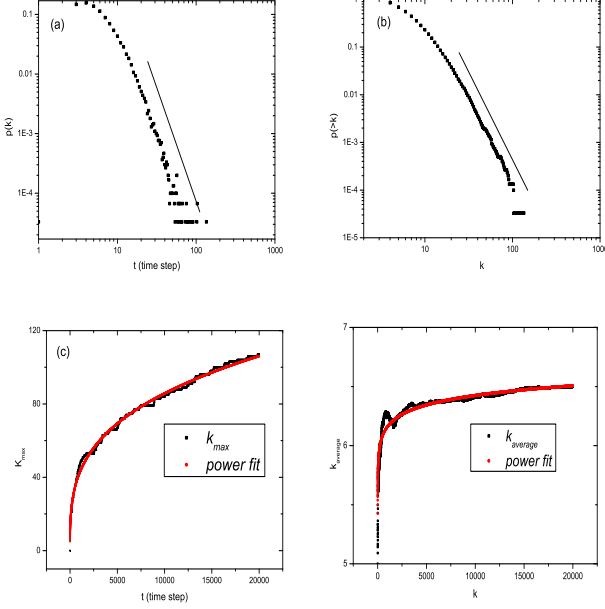
FIG. 2: The statistical and evolutive characterization of degree, where $m = 3, p = 0.06, G = 2, N = 2 \times 10^4$: (a)degree distribution $p(k)$, (b) normalized accumulative degree distribution $P(k) = \int_k^\infty kp(k)dk/k$, (c)time evolution of maximum degree $k_{max}$ and (d)average degree $k_{average}$. Obviously, $p(k) \propto k^{-\gamma}$, where $\gamma = 2.85 \pm 0.07$. Power fitness ($a * t^b$) of $k_{max}$ and $k_{average}$ are represented, with $a = 5.06 \pm 0.06, b = 0.307 \pm 0.005$ and $a = 5.42 \pm 0.06, b = 0.018 \pm 0.001$ respectively, i.e. $k_{max} \propto N^{0.307}$, $k_{average} \propto N^{0.018}$. The number of edges of the networks $E = \frac{N * k_{average}}{2} \propto N^{1.018}$, in other words, the networks are sparse.

1. Starting with a small number ($m_0$) of nodes which are global connected with each other. At each time step, a new node $i$ is added to the networks, and connect to $m$ nodes randomly.

2. The node $i$ will link with probability $p$ to the neighbors of the nodes which connected to it at the last step. And at the next $G - 1$ steps ($G$ is a fixed integer), let node $i$ execute similar operation (as *Fig.1* shows).

For simplification, the parameters $m$, $p$ and $G$ are constant. Obviously, rule*2* is a self-organized process that impacts the topological structure of the networks. *Fig.2* shows a typical scale-free network based on our model with $N = 2 \times 10^4$ nodes. The exponent $\gamma$ is a little smaller than 3.0, which is close to empirical study. And time evolution of maximum degree $k_{max}$ and average degree $k_{average}$ are exhibited in *Fig.2c* and *Fig.2d*, respectively. It can be found that $k_{max} \propto N^{0.307}$, $k_{average} \propto N^{0.018}$, thus the total number of edges in the network is proportionable to $N^{1+0.018}$, in other words, the networks is sparse. Moreover, there is evidence to suggest that in real-world networks, e.g. World Wide Web, the average degree of nodes is increasing with time [3]. Our model
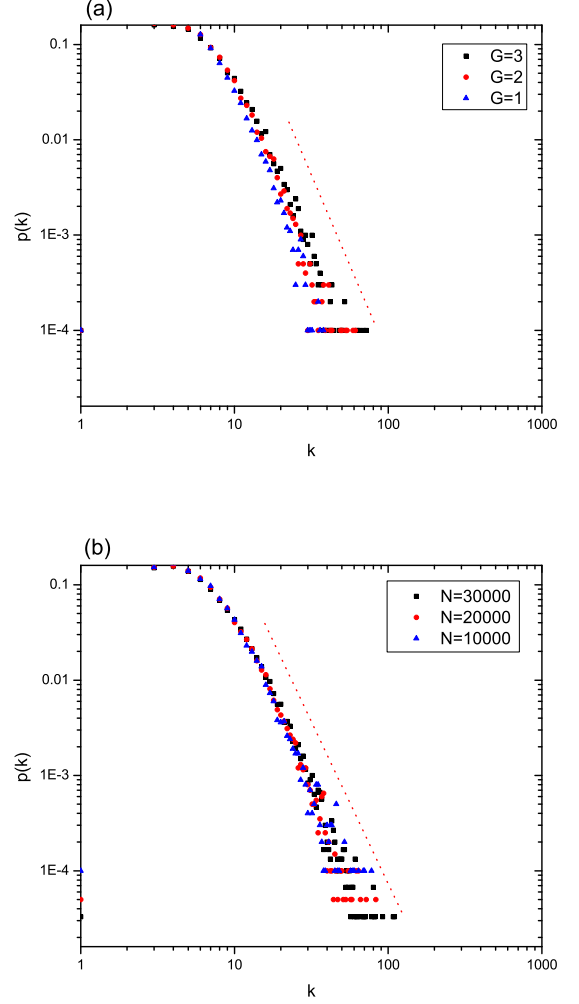




FIG. 3: Degree distribution for different $G$ and $N$: (a)$G$=1, 2, 3, $N = 10^4$; (b) $N = 10^4, 2 \times 10^4, 3 \times 10^4$, $G$=2, where $m$=2, $p$=0.06. It shows that the value of $G$ and $N$ impacts little the topology of network, i.e. different value of $G$ and $N$ leads to almost the same power-law behavior of degree distribution.

accords with that very well.

In the following we will discuss how the parameters $G$ and $p$ impact the topology of networks. In fig.3a one can see that for different $G$ the model leads to almost the same degree distribution, i.e. the value of $G$ impacts little the topology of networks. One can easily check that even for large $G$, which is close to real-world instances that people usually have wide communication with others, the networks based on our model are still of scale-free properties. This extensive result is very significant. However, when $G$ is too large (e.g. $G \geq 10$) the computer is unable to work it out. Figure 3b exhibits that $p(k)$ is independent of time and subsequently independent of network size $N = m_0 + t$, which indicates that despite its continuous growth, the network self-organizes
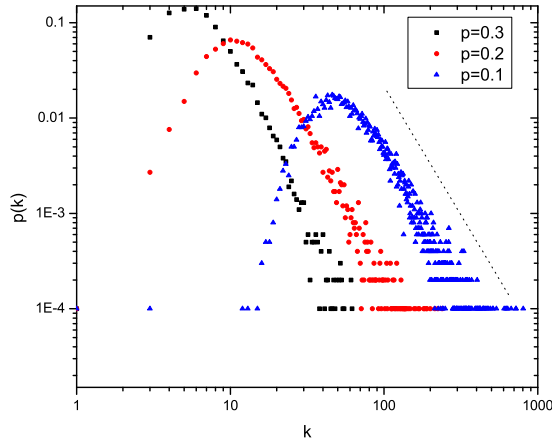
FIG. 4: The degree distribution $p(k)$ for networks with $p$=0.1, 0.2, 0.3, where $m$=2, $G$=1, $N = 10^4$. It shows that for different probability $p$, the degree distributions follows power-law behaviors $p(k) \propto k^{-\gamma}$ with almost the same exponent $\gamma = 2.90 \pm 0.09$, and for larger $p$, the rang of power-law distribution gets narrower.
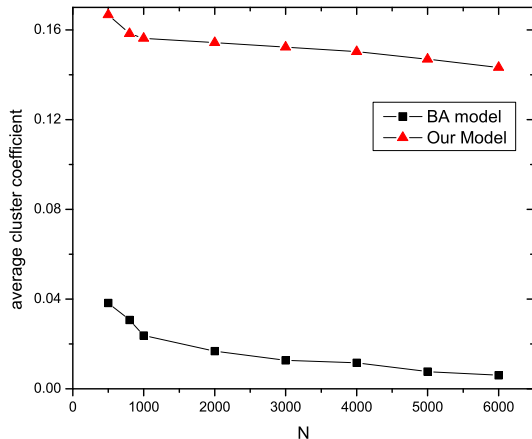


FIG. 5: Clustering coefficient of networks with different size, $N$, based on our model(triangle) and BA model(square). Obviously, the one of our model is much larger.

to a scale-free stationary state.

As illuminated above, for small values of $p$, e.g. 0.06, 0.08 *et al*, the self-organization processes induce scale-free networks. However, when $p$ is getting larger, that's to say, each node newly added to the network connects to its neighbors' neighbors with larger probability, the number of nodes with small degree will become fewer (as shown in *Fig.4*). The results indicate that the rang of power-law degree distribution will get narrower as $p$ increases. Let $p$=0, the network has exponential degree distribution. The analysis is simple. We can draw the master equation as follows:

$$N(k,t+1) = N(k,t) + \frac{N(k-1,t)}{t+m_0} \times m - \frac{N(k,t)}{t+m_0} \times m \quad (1)$$

where $N(k,t)$ is the number of nodes with degree $k$ at time step $t$. Since $N(k,t) \sim p(k)t$ and $t \gg m_0$ for large $t$, seeking solution of this form we can get that $p(k) \propto e^{-\frac{k}{m}}$. How $p$ impacts the degree distribution detailedly is significant and will be discussed in the future work.

It is well known one of the shortcomings of BA's model is that the clustering coefficient is small and decreases with the increasing of network size, following approximately a power law $C \sim N^{-0.75}$ [1]. In *Fig.5*, we report the clustering coefficient of the networks based on our model, which is much larger than that on BA's model. This is not difficult to explain. Suppose node $h1$ and $h2$ are neighbors of a common node $h3$, then according to rule*2* mentioned above, node $h1$ will link to node $h2$ with probability $p$ if the former one is added to the network earlier than the later one. This induces that the edges among neighbors of node $h3$ will become more, thus the clustering coefficient will get larger.

In summary, we propose a new model, based on "randomly attachment" and self-organization, which leads to a scale-free degree distribution of networks. The mechanism is different from preferential attachment mechanism where a scale-free distribution are generated during gradual growth of hubs. To investigate the "randomly attachment" mechanism is very important, since there are some networks that appear to have power-law degree distributions, but for which preferential attachment is clearly not an appropriate model[17, 19, 30, 31]. The simulation results exhibit that, rather than BA networks, the networks generated from our model are of larger clustering, which is close to the real-life networks. Furthermore, we suggest that self-organizing process plays a major role in many real-life networks such as collaboration networks, social networks of acquaintance or other relations between individuals and so on. We believe that many useful scale-free networks can be constructed using our approach.

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